General Relativity and Cosmology Problems

**Problem 1**
Given that $R_1^T R_1 = I$ and $R_2^T R_2 = I$, show that $R_1 R_2$ is a rotation.

**Problem 2**
If $R_{ij} = \delta_{ij} + \epsilon_{ij}$ show that $\epsilon_{ij} = -\epsilon_{ji}$.

**Problem 3**
Show that if $\epsilon_{ij} = -\epsilon_{ji}$ and $P_{ij} = P_{ji}$ then $\epsilon_{ij} P_{ij} = 0$.

**Problem 4**
Show that $[\epsilon_i, \epsilon_j] = \epsilon_{ijk} \epsilon_k$ and $\tau_i \equiv \frac{\epsilon_i}{i}$ implies that,

$$[\tau_i, \tau_j] = i\epsilon_{ijk} \tau_k.$$  

**Problem 5**
Show that the transformations that have the property $det R = 1$ form a subgroup of the $N$ dimensional rotation group.

**Problem 6**
Show that $\tau^2 \equiv \tau^2_i = 2I$. Note that this relation is of the form $l(l+1)I$ where $l = 1$, that is this is the spin one representation.

**Problem 7**
Show that all transformations of the form $R \otimes R \otimes R \otimes \cdots \otimes R$ where there are $p$ of these and where $R^T R = 1$, form a group.

**Problem 8**
Show that: $\epsilon^i_{ijk} = (det R) \epsilon_{ijk}$.

**Problem 9**
Show $L = (r \times p) = \epsilon_{ijk} r_j p_k$ is a pseudo-tensor of rank one.