Math exam, nationwide, Baccalaureate june 2003, “real” track
(science-oriented high schools)

All problems are required. Exam time is 3 hours. You have 10 points from the start - for taking the exam- (total: 100 pnts). Each question is 3 points.

Consider \((a_n)_{n \in \mathbb{N}^*}\) and \((b_n)_{n \in \mathbb{N}^*}\),

\[ a_n = \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n}; \quad b_n = a_n + \frac{1}{2n \cdot 2n^2}; \quad (\forall) n \in \mathbb{N}^* \tag{1} \]

1. The set \(\{n \in \mathbb{N}^*| a_n < a_{n+1}\}\) is:
   a) Made up of a single element;
   b) \(\mathbb{N}^*\)
   c) \(\emptyset\)
   d) Finite, having at least two elements.

2. The set \(\{n \in \mathbb{N}^*| b_n > b_{n+1}\}\) is:
   a) \(\mathbb{N}^*\)
   b) Finite, having at least two elements;
   c) Made up of a single element
   d) \(\emptyset\).

3. Knowing that \((a_n)_{n \in \mathbb{N}^*}\) and \((b_n)_{n \in \mathbb{N}^*}\) are convergent, denote by \(a = \lim_{n \to \infty} a_n\) and \(b = \lim_{n \to \infty} b_n\). Then \(a - b\) is:
   a) 0.5
   b) 0
   c) 0.25
   d) 1

4. The number \(a = \lim_{n \to \infty} a_n\) belongs to the set:
   a) \(\mathbb{Q} \setminus \mathbb{Z}\)
   b) \(\mathbb{Z} \setminus \mathbb{N}\)
   c) \(\mathbb{R} \setminus \mathbb{Q}\)
   d) \(\mathbb{N}\) \(\tag{2}\)

In the cartesian system of coordinates \(xOy\) consider the points \(A_n(n, n^2), n \in \mathbb{N}\).

5. The slope of the straight line \(A_0A_1\) is:
   a) 2
   b) 1
   c) -2
   d) -1

6. The equation of the straight line \(A_0A_1\) is:
   a) \(y = x^2\)
   b) \(y = x\)
   c) \(x^2 + y = 0\)
   d) \(x + y = 0\) \(\tag{3}\)

7. The length of the segment \(A_1A_2\) is:
   a) 10
   b) \(\sqrt{10}\)
   c) 3
   d) 4

8. The area of the triangle \(A_nA_{n+1}A_{n+2}, n \in \mathbb{N}\) is:
   a) 2
   b) \(n\)
   c) 1
   d) \(n+1\)

9. The number of straight lines that pass through 2 points in the set \(\{A_1, A_2, A_3, A_4, A_5\}\) is:
   a) 9
   b) 8
   c) 10
   d) 20

10. How many triangles have the vertices in the set \(\{A_1, A_2, A_3, A_4, A_3\}\) ?
    a) 15
    b) 5
    c) 20
    d) 10

Consider the function \(f : \mathbb{R} \to \mathbb{R}, f(x) = \sin x\). Denote by \(f^{(n)}(x)\) the n-th order derivative of the function \(f\), taken at point \(x\).
11. Which of the following numbers is the period of the function $f$?

\begin{align*}
a) \pi & \quad b) \pi/2 \quad c) 2\pi \quad d) 3\pi \\
\end{align*}

12. How many points of local maximum does the function $f$ have in the interval $[0, 11\pi]$?

a) 11 \quad b) 5 \quad c) 6 \quad d) 10

13. The area of the plane surface between the graph of the function $f$, the $Ox$ axis and the straight lines of equations $x = 0$ and $x = 2\pi$ is:

a) 0 \quad b) 2 \quad c) 4 \quad d) 3

14. The value of

$$\lim_{x \to \infty} \frac{\int_0^x |f(t)|\,dt}{x}$$

is:

a) 0 \quad b) $\frac{2}{\pi}$ \quad c) 1 \quad d) $\infty$

15. The maximum length of an interval included in $[0, 2\pi]$ on which the function $f$ is convex, is:

$$a) \frac{\pi}{2} \quad b) \pi \quad c) 2\pi \quad d) \frac{3\pi}{2}$$

16. The value $f^{(2004)}(0)$ is:

a) 1 \quad b) -1 \quad c) 0 \quad d) 0.5

Consider the polynomial $f = x^4 - 14x^2 + 9$, with roots $x_1, x_2, x_3, x_4 \in \mathbb{C}$, the number $a = \sqrt{2} + \sqrt{5}$ and the sets $A = \{g(a)|g \in \mathbb{Z}[X]\}$, $B = \{g(a)|g \in \mathbb{Z}[X], \text{rank}(g) \leq 3\}$.\n
17. Which of the following numbers is not a root of the polynomial $f$?

$$a) -\sqrt{2} + \sqrt{5} \quad b) \sqrt{2} + \sqrt{5} \quad c) \sqrt{2} - \sqrt{5} \quad d) \sqrt{2} + \sqrt{3}$$

18. The sum $x_1 + x_2 + x_3 + x_4$ equals:

a) 4 \quad b) 14 \quad c) 0 \quad d) -14

19. The product $x_1x_2x_3x_4$ equals:

a) 0 \quad b) 14 \quad c) -9 \quad d) 9

20. If $p\sqrt{2} + q\sqrt{5} + r\sqrt{10} + s = 0$, $p, q, r, s \in \mathbb{Q}$, then the value of the expression $2p + 5p + 10r + s$ equals:

a) 0 \quad b) 5 \quad c) 7 \quad d) 2

21. The set $A \setminus B$ is:

a) $\varnothing$ \quad b) Infinite; \quad c) Finite, having at least two elements; \quad d) Made up of a single element

Consider the matrices

\begin{align*}
A & \in M_{3,4} (\mathbb{C}), \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\end{align*}

22. The rank of matrix $A$ is:
23. The solution of the system of equations

\[ \begin{align*}
x + y + z + t &= 1 \\
y + z + t &= 0 \\
z + t &= 0
\end{align*} \]  

where \((x, y, z, t) \in \mathbb{C}^4\), is:

a) \((-1, 1, -1, 1)\)  
b) \((1, 1, -1, -1)\)  
c) \((1, 0, \lambda, -\lambda), \lambda \in \mathbb{C}\)  
d) \((1, -1, 1, -1)\)

24. The equation \(AX = I_3, X \in M_{3,4}(\mathbb{C})\) has

a) No solutions;  
b) An infinity of solutions;  
c) A single solution;  
d) An infinite number of solutions strictly larger than 1.

25. The matrix \(I_3A\) has the sum of its elements equal to:

a) 9  
b) 10  
c) 0  
d) 12

26. The set \(\{Y \in M_{4,3}(\mathbb{C}) | det(YA) \neq 0\}\) is:

a) Empty;  
b) Made up of a single element;  
c) Made up of a finite number of elements, at least 2;  
d) Infinite

27. How many solutions has the equations \(3x = \hat{0}\) in the ring \(\mathbb{Z}_6\)?

a) 1  
b) 4  
c) 2  
d) 3

28. The sum \(\hat{1} + \hat{2} + \hat{3} + \hat{4} + \hat{5}\), calculated in the ring \(\mathbb{Z}_6\), is:

a) \(\hat{3}\)  
b) \(\hat{2}\)  
c) \(\hat{0}\)  
d) \(\hat{1}\)

29. The product \(\hat{1} \cdot \hat{2} \cdot \hat{3} \cdot \hat{4} \cdot \hat{5}\), calculated in the ring \(\mathbb{Z}_6\), is:

a) \(\hat{1}\)  
b) \(\hat{2}\)  
c) \(\hat{3}\)  
d) \(\hat{0}\)

30. What is the order of the element \(\hat{2}\) in the group \((\mathbb{Z}_6, +)\)?

a) 2  
b) 4  
c) 6  
d) 3