I. A particle of mass \( m \) is moving under the influence of a potential, \( U(x) \), where

\[
U(x) = \frac{k}{4} \left( -x^2 + \left( \frac{2}{3a} \right) x^3 \right),
\]

where \( k > 0 \) and \( a > 0 \).

(a) Sketch the potential diagram. Find two possible equilibrium positions. Which one is stable?

(b) Find the frequency of small oscillations about the stable equilibrium.

II. An object of mass \( m \) is moving in the \( x \)-direction through a resistive medium where the resistive force varies with its velocity as:

\[
F(v) = -mkv^3,
\]

where \( k > 0 \). At \( t = 0 \), \( x(0) = 0 \) and \( v(0) = v_0 > 0 \).

(a) Find \( v(t) \) for \( t > 0 \).

(b) What is the limiting velocity as \( t \to \infty \)?

III. A particle of mass \( m \) is moving in the \( x - y \) plane, acted on by a restoring force in the \( y \) direction. The Lagrangian would be

\[
L = \left( \frac{m}{2} \right) \left( \dot{x}^2 + \dot{y}^2 \right) - m\omega_0^2 y^2,
\]

except that the motion is subject to a constraint, \( x(t) = y(t) - \left( \frac{2A}{\omega^2} \right) \sin \omega t \).

(a) Write down the Euler-Lagrange equation for \( y(t) \).

(b) The Euler-Lagrange equation obtained above can be solved by a method which you should be familiar with. If \( y(0) = 0 \), and \( \dot{y}(0) = 0 \), find \( x(t) \) for \( t > 0 \).